NUMERICAL SOLUTIONS FOR HEAT FLOW IN ADHESIVE LAP JOINTS[†]

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INTRODUCTION

The detection of disbonds in riveted lap joints is of increasing interest to the aerospace community. Adhesively bonded and riveted lap joints are used to bond the thin overlapped sheets of aluminum which comprise the outer skin of an aircraft. Through time, the integrity of the bond can become compromised by disbonding, leading to corrosion and stress concentrations at the rivets and subsequent cracking leading to joint failure. A thermal technique for determining bond integrity in these structures has been investigated by Winfree, et al [1]. This technique involves active heating of the aircraft fuselage with a measurement of the temperature on the outer surface of the structure with an infrared imager. By even application of heat to the outer surface of the lap joint, details of the inner structure become thermographically detectable. A disbond will prevent heat from penetrating from the surface layer to the subsurface layers, resulting in an increase in surface temperature over the disbond. Thermographic detection of disbonds excels over other methods by being a noncontacting, quantitative method for inspecting large areas in a short period of time.

Determining the limits of the thermal technique and optimizing the analysis and data gathering process is especially suited to simulation studies. Simulations are a cost effective method for determining the shapes and sizes of detectable disbonds for a given technique. Different data analysis techniques can be compared with controlled conditions and variations in a manner difficult to reproduce in the laboratory. Standard finite element methods (FEM) discretize the domain and seek solutions to the governing equations within these discrete elements. The thickness of the aircraft skin compared to the lateral dimensions is vastly different. An immediate problem for finite element models of this structure is the large aspect ratio cells. This decreases the convergence rate to steady state. A grid with a reduced aspect ratio generates more elements, with greatly increased the solution time. For three dimensional simulations, the increase in cpu time is especially prohibitive, although certainly not impossible,

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providing a relatively fast machine is available. Results of standard three dimensional finite element models of this geometry with various disbond sizes are presented elsewhere in these proceedings [2], [3], [4].

Other approaches, including grid clustering or patched grids, around the region of overlap can alleviate the mesh difficulties around areas of largely differing grid requirements, but are somewhat more complex than is needed for this application. Therefore, for problems with a very large aspect ratio geometry, a solution technique was sought which would eliminate the difficulties associated with the large aspect ratio grids. For this particular problem, it is possible to take advantage of the short thermal time constant of the aluminum skin. In a short period of time the thermal response through the thickness arrives at a quasi-static state. That is, the time dependent part of the solution through the thickness dies away very quickly, leaving only transverse heat flow to simulate. The dimensionality of the resulting differential equation is reduced by one, and the aspect ratio problem removed. The resulting partial differential equation can then be solved simply with standard FEM. Obviously, the cpu time for this quasi-static solution decreases dramatically compared to standard finite element solutions, especially for three dimensional problems.

THEORY

Domain Discretization

General finite element formulations are an approach to numerically solving the governing equations representing a physical system. The domain of interest, the geometry through which a physical system is to be studied, is first divided into elements in which a discrete solution to the governing equations will be sought. Some general rules for the method in which a domain is discretized can be found in references [5] and [6].

In order to characterize the heat flow with any accuracy around the disbond, the domain must be sufficiently discretized in that region. In the far field the heat flow is fairly constant and a sparse grid is sufficient. The problem is illustrated in Figure 1. A large grid aspect ratio decreases the convergence rate to the final solution due to the large difference in the magnitude of the thermal time constants in each direction. This also causes a systematic error in the solution. Even if only one element were used through the thickness of the material, the aspect ratio is still much larger than generally acceptable. Furthermore, temperature resolution through the thickness of the material is not necessary, since the transient portion of the solution decays very quickly.

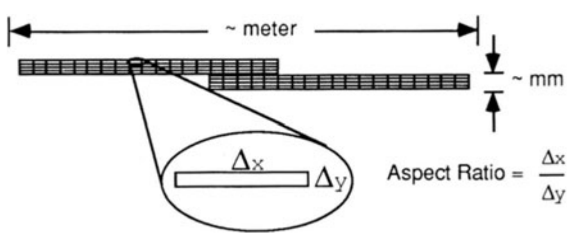

Figure 1. Illustration of aspect ratio problem for adhesively bonded lap joints in aircraft aluminum. The ratio in grid size of the transverse and through-the-thickness directions is very large, increasing the time to convergence.

Transient Decay

An appropriate model for the heat flow in a region of the lap joint is the one dimensional solution to the heat equation through a slab. Boundary conditions for this model include a constant heat flux input on the surface, an adiabatic back surface, and zero initial temperature. This solution can be written as [7]

$$T(y,t) = \frac{F\,t}{\rho c l} + \frac{F\,l}{K}\left\{ \frac{3y^2 - l^2}{6\,l^2} - \left[\frac{2}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \underbrace{e^{-\alpha n^2 \pi^2 t / l^2}}_{\text{[Transient Decay]}} \cos\frac{n\pi y}{l} \right] \right\} \quad (1)$$

Where T = Temperature
F = Flux
l = Thickness
y = Dimension through the thickness
K = Thermal Conductivity
ρ = Density
c = Specific Heat
α = Diffusivity
t = Time

Even in a "worst case" scenario, where the thickness of the aluminum is double that of the specifications for aircraft structures (as might be the case over reinforced or repaired areas), the transient portion of equation (1) decays within 25 msec (see Figure 2.). Since the infrared camera can collect an image at the rate of 30 frames per second, the transient response through the thickness of the aluminum can be neglected.

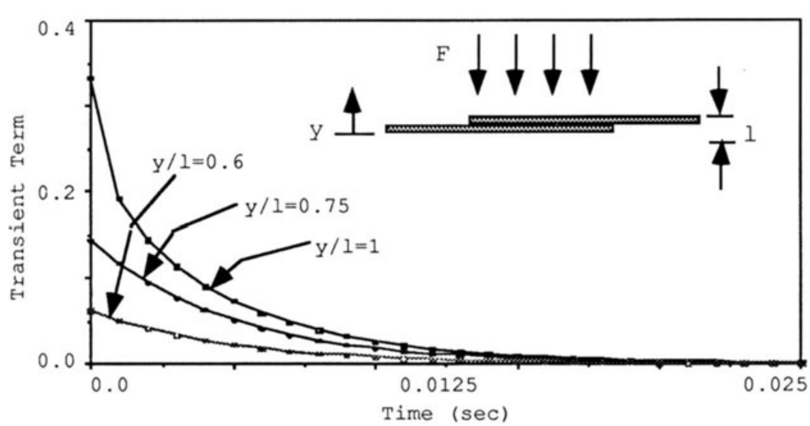

Figure 2. Transient term through the thickness of the aluminum decays rapidly due to the combination of the high diffusivity of the material and the thin layer of the structure.

Quasi-2d Solution

Assuming the transient term is negligible, a solution for the temperature in the structure is sought of the form

$$T(x,y,t) = T(x,t) + \frac{F\,l}{K}\left\{\frac{3\,y^2 - l^2}{6\,l^2}\right\} \qquad (2).$$

Substituting this into the heat equation for an isotropic material gives,

$$\alpha\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t} - \frac{F(x)\,\alpha}{Kl} \qquad (3).$$

Equation (3) is then the governing partial differential equation to be solved using standard finite element methods. Written in implicit finite element form, this equation reduces to

$$\Sigma\left(\alpha A_n' \int \frac{\partial u_m \partial u_n}{\partial x\ \partial x}dx - \int \frac{u_m u_n}{\alpha\Delta t}dx\right) = \Sigma - \int u_m u_n \frac{A_n}{\Delta t}dx - \int u_m \frac{F(x)\,\alpha}{Kl}dx \qquad (4),$$

where A_n is the discrete solution for the temperature and A_n' is the discrete solution at the next time step. The basis function, $u(x)$, is the interpolating polynomial across the element. The last integral in equation (4) is the flux term, and each layer of the lap joint can now be treated separately and coupled together through the flux term.

Domain Coupling

Equation (4) is applied to each layer separately, decoupling the solution to the full lap joint problem into two separate domains, each of which are easily solved. The heat diffusion to the second layer is incorporated through the flux term from equation (4) for the final solution. In this manner, the resistivity of the bonding agent, air, oil, water or corrosion products between the layers of aluminum that would effect the thermal signal are easily incorporated into the analysis. The flux flowing from the top layer to the bottom layer is illustrated in figure 3 and is written

$$F(x) = \frac{T_1 - T_2}{R} \qquad (5),$$

where R is the thermal resistivity between the laminates.

Writing the temperature in each layer in finite element form, the flux term from equation (4) becomes

$$\text{Flux} = -\frac{1\alpha}{RK1}\left[\sum_{n=1}^{npts_1}\int u_m(x)\ u_n(x)A_n'(t)\ dx - \sum_{n=npts_1+1}^{npts_1+npts_2}\int u_m(x)\ u_n(x)A_n'(t)\ dx\right] \qquad (6),$$

where $n \le npts1$ refers to nodes on one layer of aluminum, and $n > npts1$ refers to nodes on the second layer of aluminum as illustrated in Figure 3.

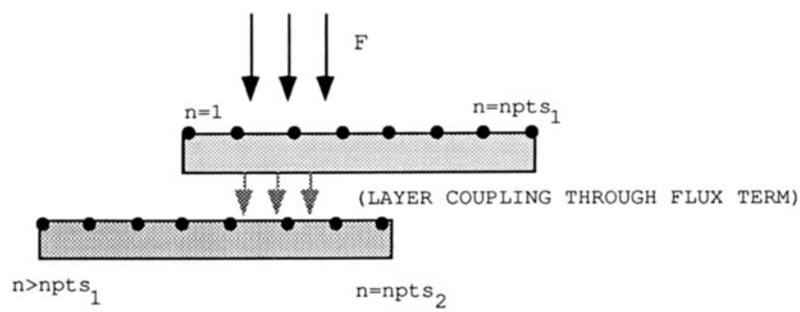

Figure 3. Layers of lap joint are coupled through the flux term between the two layers. The thermal resistivity of the coupling media, whether air, epoxy, or water, is thus easily incorporated.

RESULTS

Finite element solutions to this quasi-2d formulation were first tested against an analytic solution. For a one dimensional multilayered system, an analytic solution for the Laplace transform of the temperature can be found using a formulation such as that presented in Carslaw and Jaeger [8]. The time domain solution is found by numerically performing the inverse Laplace transform. This corresponds to the early time thermal response at the center of the lap joint. The good agreement between this solution and that found for the equivalent finite element representation is shown in Figure 4. A small discrepancy exists in the first 0.25 sec, which may be due to the time steps taken in the finite element calculations. Certainly by 0.5 sec, the models are essentially identical, and the overall temperature rise of the sample is consistent with a lap joint configuration. This indicates that the methodology accurately represents the coupling of the two layers.

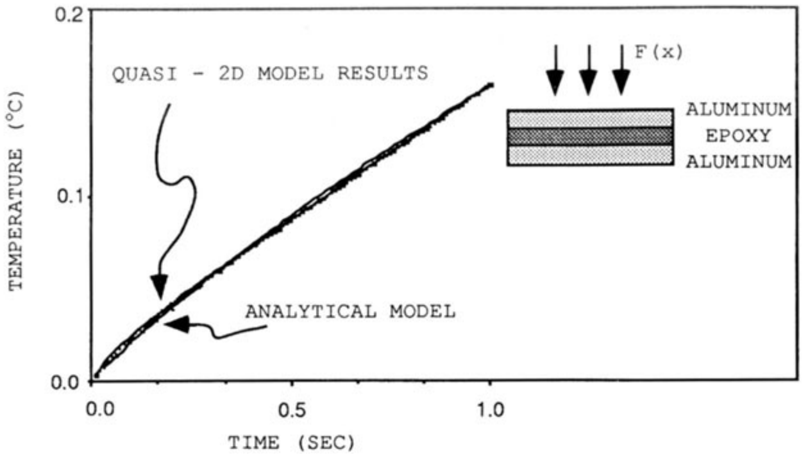

Figure 4. Comparison of quasi-2d finite element results with an analytic solution for flux input into a layered media.

The quasi-2d solution to the lap joint configuration was next compared to results obtained with standard finite element techniques. The samples were 21.5 cm wide sheets of aluminum overlapped by 7.5 cm and bonded together with epoxy having a thermal conductivity of 5e-3 cal/s/cm/°C as shown in Figure 5. Temperature profiles across the front surface of the sample show very good agreement between the two simulations (see Figure 6.), however some discrepancy is observed over the right hand side of the lap joint. Furthermore, the standard deviation of this error increases with time. Decreasing the node spacing removes this discrepancy. These temperature profiles are not presented here due to space limitations. Errors associated with the aspect ratio of the grid in standard finite element formulations are thus eliminated in this quasi-2d formulation, without the penalty of largely increasing computational time.

Incorporation of the disbond in the formulation is relatively simple. The coupling term across the domain is adjusted across that region of the lap joint that is disbonded. Figure 7 illustrates the temperature profile over a disbonded region compared to a completely bonded region.

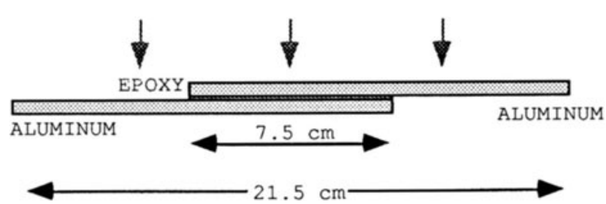

Figure 5. Lap joint geometry used for comparison of standard finite element simulations and quasi-2d simulation results.

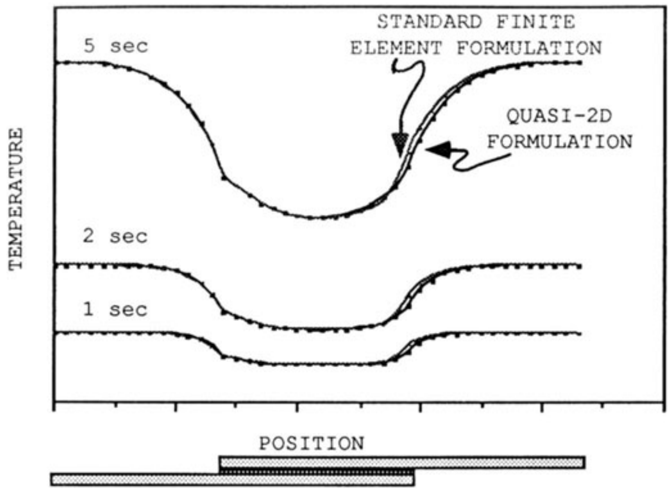

Figure 6. Comparison of resulting temperature profiles for standard finite element formulations and the quasi-2d formulation presented herein.

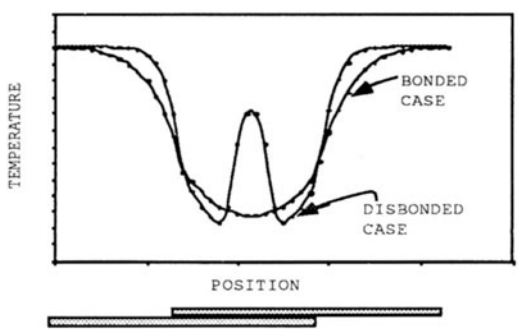

Figure 7. Temperature profiles over a lap joint for both bonded and disbonded cases.

CONCLUSIONS

A formulation for modeling heat transfer in thin, adhesively bonded lap joints has been presented, where the heat flow through the thickness of the plates is approximated as quasi-static. This approximation removes the difficulties associated with large aspect ratio grids necessary for standard finite element formulations. The resulting partial differential equation is solved using finite element analysis. This quasi-static formulation also reduces the dimensionality of the problem by one, reducing computational requirements, which is especially important for three dimensional models. The solutions are found to be in good agreement with analytical solutions and solutions from standard finite element programs. Disbonded regions of the lap joint are easily incorporated into the formulation. This approach makes possible a more accurate representation of changes in heat flux between the layers due to a disbond, and provides for a method whereby optimum heating times and heating protocols for various disbond shapes and sizes can be determined in reasonable computational time.

REFERENCES

1. W. P. Winfree, B. Scott Crews, Hazari Syed, and Patricia Howell, Proceedings of the 37th International Instrumentation Symposium, pp. 1097-1105 (1991)

2. W. P. Winfree, B. Scott Crews, and P. A. Howell, "Comparison of Heating Protocols For Detection of Disbonds in Lap Joints," in Review of Progress in Quantitative NDE, edited by D. O. Thompson and D. E. Chimenti (Plenum Press, New York, 1992)

3. D. R. Prabhu, W. P. Winfree, "Automation of Disbond Detection in Aircraft Adhesive Joints Through Thermal Image Processing," in Review of Progress in Quantitative NDE, edited by D. O. Thompson and D. E. Chimenti (Plenum Press, New York, 1992)

4. D. R. Prabhu, P. A. Howell, H. I. Syed, and W. P. Winfree, "Application of Artificial Neural Networks to Thermal Detection of Disbonds," in Review of Progress in Quantitative NDE, edited by D. O. Thompson and D. E. Chimenti (Plenum Press, New York, 1992)

5. Joe F. Thompson, Z. U. A. Warsi, and C. Wayne Mastin, Numerical Grid Generation, Foundations and Applications (North-Holland, New York, 1985)

6. Numerical Grid Generation Techniques, NASA Conference Publication 2166, Scientific and Technical Information Office (1981)

7. H. S. Carslaw and J. C. Jaeger, Conduction of Heat in Solids p. 112 (Clarendon Press, Oxford, 1988)

8. Carslaw and Jaeger pp. 319-326